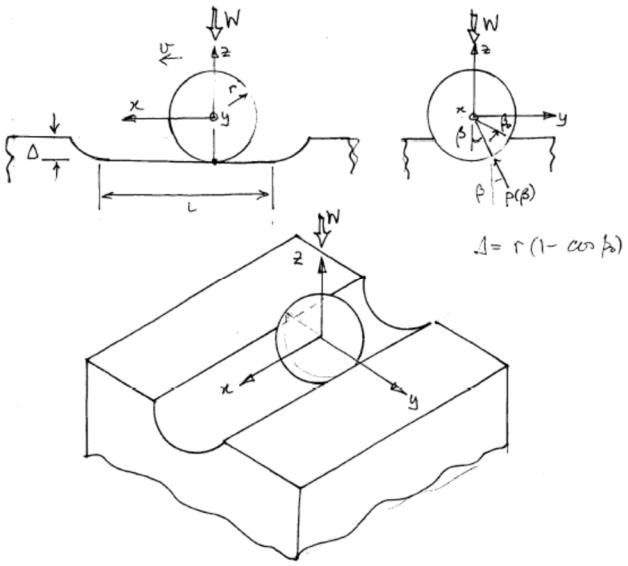
Guidance – Hard Ball on Soft Flat Contact Pressure Change

Detailed Analysis by Professor John Williams

Consider a hard ball radius *r*, reciprocating in a straight line on a softer, in other words, wearing surface. The depth of wear is Δ and the sliding speed is *v* and the applied load is *W.*

Viewed from the side, normal to the x-z plane, the loading can be considered to be concentrated in a line (rather than a thin ellipse), but viewed normal to the sliding direction there is conformity between ball and track. The semi-included angle is, say, β_0:

Geometry: $\Delta = r(1 - \cos\beta_0)$ (1)



Now, the contact pressure, thought of as the intensity of the line loading, will vary along the arc of contact so that $p \approx p(\beta)$. But equilibrium must be satisfied so that:

$$\int_{-\beta_0}^{+\beta_0} p(\beta)\cos\beta\, r\, d\beta = W \qquad (2)$$

Now suppose as the process proceeds and the depth of the wear track increases it does so in such a way that the Archard wear law is satisfied at every point. If the centre line wear is δΔ then at the position β the radial wear must be δΔ/Cos β is the ball is to remain spherical. So applying Archard at these two point and calling the intensity of loading on the centre line p_0:

$$\frac{d\Delta}{dt} = K_w p_0 v \quad \text{and} \quad \frac{d\,\Delta/\cos\beta}{dt} = K_w p(\beta) v \qquad (3)$$

So it follows that:

$$p(\beta) = p_0 \sec\beta \tag{4}$$

Thus applying equilibrium (2):

$$2\int_0^{+\beta_0} p_0 \sec\beta \cos\beta \, r \, \mathrm{d}\beta = W \quad \text{i.e.} \quad p_0 = \frac{W}{2r\beta_0}$$

And so:

$$p(\beta) = \frac{W}{2r\beta_0}\sec\beta \tag{5}$$

So the wear equation (3) becomes:

$$\frac{\mathrm{d}\Delta}{\mathrm{d}t} = K_w p_0 v = K_w \frac{W}{2r\beta_0} v \tag{6}$$

Differentiating equation (1) we have:

$$\frac{\mathrm{d}\Delta}{\mathrm{d}t} = r\sin\beta_0 \frac{\mathrm{d}\beta_0}{\mathrm{d}t} \tag{7}$$

So, equating the right hand sides of (6) and (7), we have:

$$r\sin\beta_0 \frac{\mathrm{d}\beta_0}{\mathrm{d}t} = K_w \frac{W}{2r\beta_0} v \tag{8}$$

But call the nominal pressure on the contact $P = W/\pi r^2$ and remembering that the integral of v with time must be the total sliding distance L, we can write (8) as:

$$\int_0^{\beta_0} \beta_0 \sin\beta_0 \, d\beta_0 = \frac{\pi}{2} K_w P L \tag{9}$$

Now, $\int x \sin x \, dx = \sin x - x \cos x$, so we have an expression relating the growth of angle β_0 with applied load P, distance L and wear constant K_w:

$$\sin\beta_0 - \beta_0 \cos\beta_0 = \frac{\pi}{2} K_w P L \tag{10}$$

Now, if angle β_0 is relatively small (our data suggests typical values not exceeding 10°) then we can use the small angle approximations for sine and consine: $\sin\theta \approx \theta - \theta^3/6$ and $\cos\theta \approx 1 - \theta^2/2$, giving:

$$\beta_0 \simeq \left[\frac{3\pi}{2} K_w P L\right]^{1/3} \tag{11}$$

We can now substitute this in the expression for the linear wear Δ to get:

$$\frac{\Delta}{r} \simeq \frac{1}{2}\left[\frac{3\pi}{2} K_w P L\right]^{2/3} \rightarrow 1.4\left(K_w P L\right)^{2/3} \tag{12}$$

Or, in terms of wear volume, since the cross-sectional area of the groove is $r^2(\beta_0 - \sin\beta_0\cos\beta_0)$ and again using the small angle approximations:

$$\frac{V}{r^2 L} \approx \pi\left(K_w P L\right) \tag{13}$$

In other words, the volume wear is linear with sliding distance and load, the reduction of load intensity being compensated by the increase in the load area. It would be interesting to see if there is any evidence that the linear wear rate goes with sliding distance to the 2/3 and volume with distance.

Experimentally, one would normally measure the width of the track, say $2a$, but since β_0 is small:

$$a \approx \sqrt{\Delta \times 2r} \quad \text{i.e.} \quad a \approx r\sqrt{2\frac{\Delta}{r}} \quad \text{so from (12)} \quad a \approx 1.67r\left(K_w PL\right)^{1/3} \tag{14}$$

The data for a 6 mm ball on soft plate with PAO suggest $P \approx 28/(\pi \times 0.003^2) = 0.99$ MPa and after 1739 cycles L = 1739 x 0.05 = 87 m, with a = 380 microns. Hence, from (14):

$$Kw = 5 \times 10^{-12}\ m^2N^{-1}$$

$$= 5 \times 10^{-3}\ mm^3N^{-1}m^{-1}$$

Pin on disc data from Archard and Hurst suggest for mild steel on tool steel $Kw = 7 \times 10^{-3}\ mm^3N^{-1}m^{-1}$; this is surely too close to be a coincidence!